Friday

Chapter 7.2 Day 1

Hypothesis Testing for the Mean (σ known)

Example: The P-value for you test statistic is P = 0.1035
What should you do if your α = 0.05?

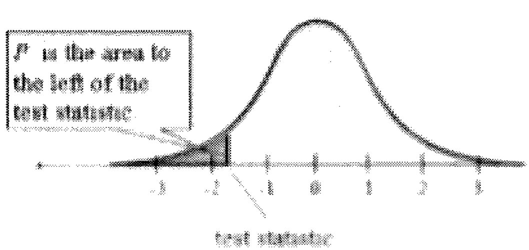
.1035 > .05
fail to reject Ho

Decision Rule Based on P-value

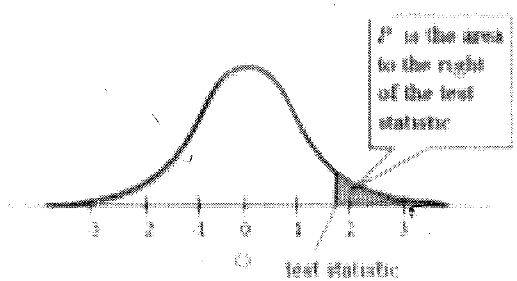
- Compare the P-value with α.
  - If $P \le \alpha$, then reject H_0.
  - If $P > \alpha$, then fail to reject H_0.

HYPOTHESIS TESTS OF μ, GIVEN X IS NORMAL AND σ IS KNOWN

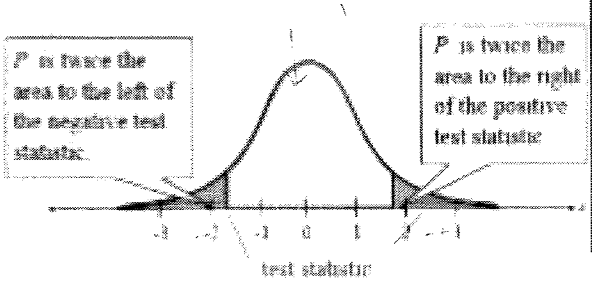
LEFT TAILED TEST:



RIGHT TAILED TEST:



TWO-TAILED TEST:



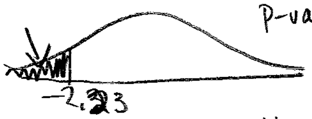
Using P-values for a z-Test for Mean μ

In Words	In Symbols
1. Verify that σ is known, the sample is random, and either the population is normally distributed or $n \ge 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Find the standardized test statistic.	$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$
5. Find the area that corresponds to z.	*Standard Normal Distribution Table.*
6. Find the P-value. a. *left-tailed test*, P = (Area in left tail). b. *right-tailed test*, P = (Area in right tail). c. *two-tailed test*, P = 2(Area in tail of standardized test statistic).	
7. Make a decision to reject or fail to reject the null hypothesis.	If $P \le \alpha$, then reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Finding a P-value

Example 2: Find the P-value for the hypothesis test with the standardized test statistic z. Decide whether to reject the null hypothesis for the level of significance indicated.

a. Left-tailed test, $z = -2.23$, $\alpha = 0.01$

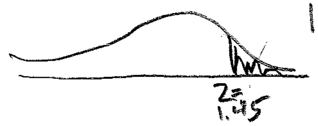


P-value = .0129 > .01

P > α

Fail to Reject Ho

b. Right-tailed test, $z = 1.45$, $\alpha = 0.1$



1 - .9265

P-value = .0735

.0735 ≤ .1

P ≤ α

Reject Ho

c. Two-tailed test, $z = 2.14$, $\alpha = 0.05$

2(.0162) = .0324

P-value = .0324 ≤ .05

Reject Ho

d) right-tailed, z = 1.45, α = .05

P-value = .0735

.0735 > .05

fail to reject Ho

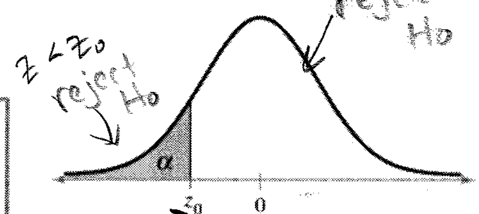
extra example

Finding Critical Values and Rejection Regions

Another method to decide whether to reject the null hypothesis is to determine whether the standardized test statistic falls within a range of values called the rejection region.

DEFINITION

A **rejection region** (or **critical region**) of the sampling distribution is the range of values for which the null hypothesis is not probable. If a standardized test statistic falls in this region, then the null hypothesis is rejected. A **critical value** z_0 separates the rejection region from the nonrejection region.

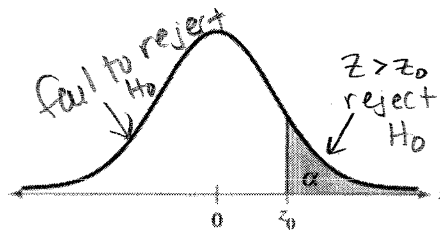


Left-Tailed Test

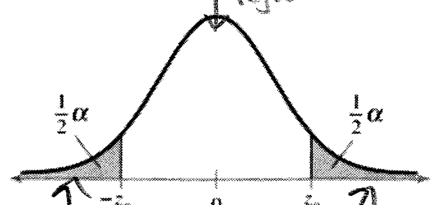
GUIDELINES

Finding Critical Values in the Standard Normal Distribution

1. Specify the level of significance α.
2. Determine whether the test is left-tailed, right-tailed, or two-tailed.
3. Find the critical value(s) z_0. When the hypothesis test is
   a. *left-tailed,* find the z-score that corresponds to an area of α.
   b. *right-tailed,* find the z-score that corresponds to an area of $1 - \alpha$.
   c. *two-tailed,* find the z-scores that correspond to $\frac{1}{2}\alpha$ and $1 - \frac{1}{2}\alpha$.
4. Sketch the standard normal distribution. Draw a vertical line at each critical value and shade the rejection region(s). (See the figures at the left.)



Right-Tailed Test



Two-Tailed Test

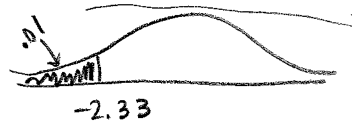
reject Ho

z < -z0 or z > z0

Example 3: Find the critical value(s) and rejection region(s) for the type of z-test with the level of significance listed. Include a graph with your answer.

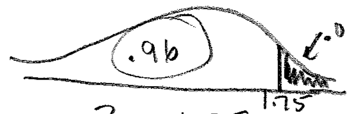
a. Left-tailed test, $\alpha = 0.01$

Crit. val: $\alpha = .01$ look up .01 inside table $\rightarrow z_0 = -2.33$ ← critical value



rejection region: $z < -2.33$

b. Right-tailed test, $\alpha = 0.04$ ←



$1 - .04 = .96$

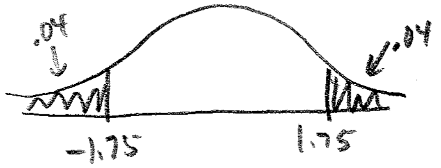
$z_0 = 1.75$ ← critical value

rejection region: $z > 1.75$

c. Two-tailed test, $\alpha = 0.08$

critical values: $z_0 = \pm 1.75$

rejection regions: $z < -1.75$ or $z > 1.75$



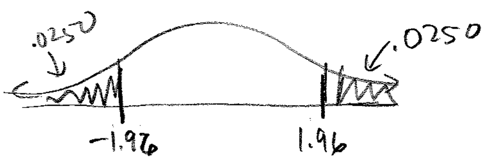
Example 4: Test the claim about the population mean at the level of significance given. Assume the population is normally distributed.

a. Claim: $\mu = 50$; $\alpha = 0.05$; $\sigma = 1.85$
Sample statistics: $\bar{x} = 49.6$, $n = 36$

$H_0: \mu = 50$ (claim)

$H_a: \mu \neq 50$

two-tailed



$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{49.6 - 50}{\frac{1.85}{\sqrt{36}}} = -1.30$$

$\alpha = 0.05 \rightarrow z_0 = \pm 1.96$

Since $-1.30 > 1.96$ and $-1.30 < 1.96$,
fail to reject H_0

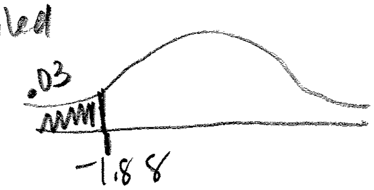
There is not enough evidence at the 5% level of significance to reject the claim.

b. Claim: $\mu < 2500$; $\alpha = 0.03$; $\sigma = 75$
Sample statistics: $\bar{x} = 2485$, $n = 121$

$H_0: \mu \geq 2500$

$H_a: \mu < 2500$ (claim)

left-tailed



$$z = \frac{2485 - 2500}{\frac{75}{\sqrt{121}}} = -2.20$$

$\alpha = .03 \rightarrow z_0 = -1.88$

Since $-2.20 < -1.88$, reject H_0

There is enough evidence to at the 3% level of significance to support the claim.